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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
UNCLASSIFIED	

DTIC
ELECTE
JUN 6 1989
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AD-A208 627

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
	AFOSR-TR- 89-0724

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
Carnegie-Mellon University		Air Force Office of Scientific Research

6c. ADDRESS (City, State, and ZIP Code)	7b. ADDRESS (City, State, and ZIP Code)
5000 Forbes Avenue Pittsburgh, PA 15213-3890	Building 410 Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	AFOSR-85-0360

8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS			
Building 410 Bolling AFB, DC 20332-6448	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
	61102F	2304	A1	

11. TITLE (Include Security Classification)

Singular and Bang-Bang Stochastic Control

12. PERSONAL AUTHOR(S)

Steven E. Shreve

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 30 Sep 85 TO 29 Sep 88		

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The proposal which formed the basis for this grant consisted of three parts: research on singular stochastic control, research on bang-bang stochastic control, and the creation of a graduate text which would make these two topics more widely accessible. Significant progress has been made in the two research areas, although a number of important questions remain. The proposed graduate text has been completed and a copy was provided to the Air Force Office of Scientific Research in December 1987. We discuss each of these three topics in turn. (KR

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
Lt. Col Crowley	(202) 767-5028	NM

DD Form 1473, JUN 86   Previous editions are obsolete.   SECURITY CLASSIFICATION OF THIS PAGE

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FINAL REPORT
SINGULAR AND BANG-BANG STOCHASTIC CONTROL
AFOSR GRANT NUMBER 85-0360
October 1, 1985 - September 30, 1988

Steven E. Shreve, Principal Investigator Department of Mathematics
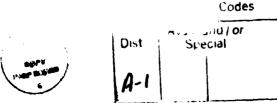Carnegie Mellon University
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The proposal which formed the basis for this grant consisted of three parts: research on singular stochastic control, research on bang-bang stochastic control, and the creation of a graduate text which would make these two topics more widely accessible.   Significant progress has been made in the two research areas, although a number of important questions remain.   The proposed graduate text has been completed and a copy was provided to the Air Force Office of Scientific Research in December 1987.   We discuss each of these three topics in turn.

1.   Singular Stochastic Control.   Singular stochastic control was designed to model the limiting behavior of a control system in which instantaneous displacements in the state can be caused by the control.   When these displacements are small and frequent, the model of singularly continuous displacement is appropriate.   Such problems were originally motivated by thruster control of spacecraft, but they have more recently been used in the study of queueing systems.

In our original proposal we identified two important problem areas in singular stochastic control.   The first was the area of finite-fuel problems, i.e., control of systems in which the maximum amount of displacement the controller is allowed to cause is some finite number known beforehand.   Such problems are in the spirit of the original application in control of spacecraft.   Our proposal was to adapt techniques we had used in the analysis of one-dimensional problems, including their reduction to problems of optimal stopping, to study finite-fuel problems in which the amount of fuel remaining constituted a second state variable.   This approach was shown to be viable in Karatzas & Shreve [1].   Work in this direction is continuing with the assistance of graduate students who have been partially supported by the grant.

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The second area of singular stochastic control in which we proposed to focus our efforts involved generalization of known results to higher dimensions. One issue we wished to address in this regard was the regularity of the value function in higher dimensions. The continuity of the second derivative of this function played an important part in the solution of a number of one-dimensional problems, but there was no theory suggesting that this high degree of smoothness would be present in higher dimensional problems. It has now been determined in joint work with Mete Soner [2] that the hoped for smoothness is present in fairly general two-dimensional problems. Our continuing efforts are directed toward establishing a similar result in more than two dimensions.

Another facet of singular control was studied by Arthur Heinricher and Victor Mizel while the former was partially supported by this grant. A deterministic control problem which exhibited the Lavrentiev phenomenon was rendered stochastic by the addition of white noise to the state equation. The Lavrentiev phenomenon occurs when the minimum cost attainable by using control laws which are Lipschitz continuous in time is strictly larger than the cost attainable by using some larger class of control laws. This has important ramifications which one considers numerical approximation schemes. When the stochastic version of a particular Lavrentiev deterministic problem was examined, it was discovered that the minimum expected cost attainable by using control processes which were absolutely continuous functions of time was strictly larger than the minimum attainable using singularly continuous processes. Approximation of singularly continuous stochastic control problems by absolutely continuous ones has been used as both a theoretical and a numerical tool, and the Heinricher-Mizel example indicates the limitations of that approach. Their work is reported in [3,4].

2. **Bang-Bang Stochastic Control.** Bang-bang stochastic control problems are those in which the drift of the controlled process changes discontinuously as the state crosses certain boundaries. Using the Girsanov change of measure theorem and Brownian local time, it is possible in some cases to compute explicitly the transition densities for processes controlled in this way. In work supported by a previous proposal, Ioannis Karatzas and I noted a remarkable coincidence involving the joint distributions of Brownian motion, its local time, occupation time, the time it attained its maximum, and the value of this maximum. We sought an explanation of this coincidence

by decomposing the Brownian path and reconstructing it so that some of these objects mapping into other of these objects. That project is now completed and is reported in [5]. The methodology of path decomposition applied here is currently receiving much attention by probabilists, and it may be that there are additional applications for control theory.

3. Brownian Motion and its Local Time. The theory of Itô processes and the application of this theory to stochastic control has reached a level of development which justifies a textbook treatment of these topics. In particular, we found the difficult and esoteric concept of Brownian local time to be an indispensible tool in dealing with a number of stochastic control problems, but we were unable to find a textbook we could recommend to our graduate students for a treatment of this and related topics. For this reason, Ioannis Karatzas and I undertook to write a graduate text on Brownian motion, stochastic integration with respect to continuous, square-integrable martingales, strong and weak solutions of stochastic differential equations, and Brownian local time. The project actually began in the fall of 1984 and was concluded with the appearance of the book in November 1987. The book has 470 pages, and in addition to the presentation of theory, contains about 275 problems and exercises, many with solutions. It is our goal to make the topics treated under this grant accessible to the mathematics, engineering and economics communities.

4. Publications Acknowledging AFOSR Grant No. 85-0360.

[1] Karatzas, I. & Shreve, S. E., *Equivalent models for finite-fuel stochastic control,* **Stochastics 18,** 1986, 245-276.

[2] Soner, M. & Shreve, S. E., *Regularity of the value function for a two-dimensional singular stochastic control problem*, **CMU Math. Dept.**
Research Report No., 88-26, Aug. 1988, submitted for publication

[3] Heinricher, A. & Mizel, V., *A stochastic control problem with different value functions for singular and absolutely continuous control*, **Proc. 25th IEEE Conf. Decision and Control, Dec. 1986, 134-139.**

[4] Heinricher, A. & Mizel, V., *A new example of the Lavrentiev phenomenon*, SIAM J. Control and Optimization, to appear Nov. 1988.

[5] Karatzas, I. & Shreve, S. E., *A decomposition of the Brownian path*, Statist. & Probab. Letters 5, 1987, 87-93.

[6] Karatzas, I. & Shreve, S. E., Brownian Motion and Stochastic Calculus, Graduate Texts in Mathematics, Springer-Verlag, New York, 1988.